**APPLIED MATHEMATICS II**

**CHAPTER-1: SEQUENCE AND SERIES**

**1.1. DEFINITION AND TYPES OF SEQUENCE**

**SEQUENCE**

A***sequence*** is a function whose domain is the collection of all integers greater than or equal to a given integer (usually *0* or *1*) where as the range may be any set **.**

**REAL SEQUENCE**

A ***real sequence*** is a function whose domain is the collection of all integers greater than or equal to a given integer (usually *0* or *1*) where as the range is a subset of the set of real numbers.For example,for, for, forare sequences.

If for , then we would write for the sequence. The symbolin is called an index and is called the initial index.

**RANGE OF A SEQUENCE**

The set of all distinct terms of a sequence is called *its range*.

**Note:** In a sequence , the number of terms of a sequence is always infinite. The range of a sequence may be a finite set. For example, if, then.The range of sequence , which is a finite set.

**CONSTANT SEQUENCE**

A sequence defined by for all is called a *constant sequence*. Thus

 is a constant sequence with, a singleton.

**BOUNDED AND UNBOUNDED SEQUENCE**

The sequence is said to be **bounded** if and only if there exists a real number such that

for each

That is, if the range of the sequence is bounded.

The sequence is said to be **unbounded** if and only if it is not bounded.

**Examples:**(a) The sequence is bounded, since for each positive integer .

 (b) The sequence is not bounded.

 (c) Every constant sequence is bounded.

**CONVERGENT SEQUENCE**

A sequence is said to converge to the real number if and only if for each , there exists a real number ( depending on) such that

.

The real number is called the limit of the sequence .

The sequence is said to be convergent if and only if there is a real number such that converges to

The sequence is said to be divergent or to diverge if and only if it is not convergent.

**Example:**By definition show that the sequence converges to .

**Solution:** Let be given, and choose a real number such that .Then for, we have . Hence

**Theorem:** Every convergent sequence has a unique limit **OR** a sequence cannot converge to more than one limit.

**Example:** Show that the sequence diverges.

**Solution:** We have .

If , then for odd values of we have But if , then for even values of we have .

Consequently, for any such that , there is no number satisfying the definition of convergent sequence.

**DIVERGENT SEQUENCE**

**(i)**A sequence is said to diverge to if given any positive real number , there exists a

positive integer (depending on ) such that

and we write

**(ii)** A sequence is said to diverge to if given any positive real number , there exists

a positive integer (depending on ) such that

and we write

**(iii)** A sequence is said to be a divergent sequence if it diverges to or .

That is, or .

**Examples:**(a) The sequences and diverge to .

1. The sequences and diverge to .

**Theorem:** Letbe a sequence,and be a function defined onsuch that

.

If , then diverges and . Thus

.

**Example:** Show that the sequence diverges.

**Solution:** Let. Then. Since, we conclude that. Hence, the sequence diverges.

**OSCILLATORY SEQUENCES**

If a sequence neither converges to a finitenumber nor diverges to or , it is called an *oscillatory sequence*. For example, the sequences and are oscillatory sequences.

**NULL SEQUENCE**

A sequence is said to be a **null sequence** if it converges to zero. That is,. For example, the sequences and are null sequences.

**MONOTONIC SEQUENCES**

1. A sequence is said to be **increasing** if and only if .

That is, .

For example, the sequencesand are increasing.

1. A sequence is said to be **decreasing** if and only if *.*

That is, .

For example, the sequencesand are decreasing.

1. A sequence which is either increasing or decreasing is called a **monotonic sequence**.

For example, the sequences , andare monotonic.

Note that a sequence need not be increasing or decreasing. For example, the sequence is neither increasing nor decreasing because it oscillates between and

**Theorem:** If is a monotone bounded sequence, then it is convergent.

* 1. **PROPERTIES OF CONVERGENT SEQUENCES**

Let and be convergent sequences. Then the sum , any scalar multiple , product and the quotient (provided ) all converge, with

**Example:** Use the above theorem to prove that the sequence is convergent and find the limit of the sequence.

**Solution:**

We see that the sequence is convergent and and also the sequence is convergent and.

Therefore, . Thus the given sequence is convergent and its limit is.

**Theorem:The SqueezingTheorem for Sequences**

If and is any sequence such that for , then converges, and moreover

.

**Examples:**

1. Show that .

**Solution:** We know that . This implies that .

.

Since and , it follows from the squeezing theorem for sequences that .

1. Show that .

**Solution:** For , we have , so that

.

Therefore,. Since and , it follows from the squeezing theorem for sequences that .

1. because.
2. because.
3. because and clearly .

**Example:** Using L’Hopital’s rule show that

1. is converges to
2. is converges to

**Solution:**

1. . Thus, is converges to .

. Thus, is converges to .

1. Let . Then .

.

Since then. Thus, converges to .

**Theorem:** (a) If converges, then is bounded.

 (b) If is unbounded, then diverges.

**Note:**(i).A bounded sequence may diverge. For example, is bounded but it is not

convergent.

(ii). Abounded monotonic sequence is convergent.

**Example:** Prove that the sequenceis convergent.

**Solution:** The elements of the given sequence are:

We see that and so the given sequence may be decreasing.If , i.e., .

If , then and if, then the sequence is decreasing and hence monotonic. Since for all , we conclude that the sequence is bounded. Thus, the sequence is bounded monotonic sequence. Therefore, it is convergent.

* 1. **SUBSEQUENCE AND LIMIT POINTS**

Let be a given sequence. If is a strictly increasing sequence of natural numbers (i.e., *,* then is called a subsequence of .

**Examples:**

1. The sequenceand areall subsequences of the sequence.
2. are all subsequences of a sequence

**Remark:**

1. If is subsequenceofandis a subsequence of , then is a subsequence of
2. Every sequence is a subsequence of itself.

A real number is called a **limit point** of a sequence if and only if every neighborhood of contains infinitely many terms of the sequence.

That is, there exists such that *.*

**Note:**

1. Limit point of a sequence is different from limit of a sequence.
2. If for for finitely many values of , then cannot bea limit point of .
3. Limit point of a sequence need not be a terms of a sequence.

For example, is a limit point of the sequence*.*

**Example:** Show that the sequence has two limit points.

**Solution:** Let , then

Thus every neighborhood of contains all the odd terms (since each ) of sequence. Therefore is a limit point of the sequence.

Also, every neighborhood of contains all the even terms (since each ) of sequence. Therefore is a limit point of the sequence.

**Example:** Show that the sequence has no limit point.

**Solution:**Let be anyreal number, then the neighborhood of contains at most one term of the sequence . Hence, is not a limit point of the sequence .

* 1. **DEFINITION OF INFINITE SERIES**

If is a sequence of numbers (complex or real), then the expression of the form

(i.e., the sum of the terms of the sequence, which is infinite) is called an **infinite series**.

The numbers are called the **terms** of the series.

The sum of the first terms is

.

This expression is called the  **partial sum** of the series.

 *.*

 *.*

 *.*

 *.*

 are the first, second, third, fourth,… partial sums of the series.Thus is called the **sequence of partial sums** of the infinite series . Therefore, to every infinite series , there corresponds a sequence of its partial sums.

**Example:** Compute the fourth partial sum for each of the following series.

1. 1. **CONVERGENCE AND DIVERGENCE, PROPERTIES OF CONVERGENT SERIES.**

**CONVERGENCE AND DIVERGENCE OF SERIES**

An infinite series converges or diverges according as the sequence of partial sums

converges or diverges.

**Definitions:**

1. The series converges (or is said to be convergent) if the sequence of its partial sums converges. Thus, is convergent if .

If , then the number is called its value or sum, and we write

.

1. The series diverges (or is said to be divergent) if the sequence of its partial sums diverges. Thus, is divergent if .

**Example:**Discuss the convergence or otherwise of the series

.

**Solution:**Here for . Putting , we have

.

Hence,

.

Thus, .

Since , it follows that the given series converges and the sum is . That is,

.

**Note: (i)** The series is called a **telescoping series** because when we write the

partial sums, all except the first and last terms cancel.

 **(ii)** The series is known as harmonic series.

**Example:** Show that the series diverges.

**Solution:** Observe that

;

;

.

In general,

.

Since , it follows that the sequence of partial sums is unbounded.

Hence, diverges.

**A DIVERGENCE TEST**

**Theorem:**If the infinite series is converges, then .

**Proof:** Let be the sequence of partial sums for and. Therefore,

 such that. Also, . Then

.

So . Therefore, .

**Note:** The converse of the above theoremis false.That is, if it does not follow that the series is necessarily convergent.In other words it is possible to have a divergent series for which . For example,, but diverges.

**Theorem:** If then the series is divergent.

**Proof**: Assume that is convergent, then by above theorem. But this contradicts the hypothesis. Therefore, the series is divergent.

**Example:** Show that the following two series are divergent.

**Solution:(a)**. Thus, the series is divergent.

**(b)**.which

 does not exist. Therefore, the series is divergent.

**GEOMETRIC SERIES**

A geometric series is a series of the form, where and are constants and . The convergence of a geometric series depends entirely on the choice of .

**Theorem (Geometric Series Theorem)**

Let be any number, and let and . Then the geometric series converges if and only if .

For *,*

 .

**Note:** The number is called the **ratio** of the geometric series. By the above theorem, the sum of a convergent geometric series is equal to the first term () divided by .

**Example:**Find the sum of the following series

1. (Ans: )
2. (Ans: )

**Remarks:**

**(i)** Not all geometric series converges. For example, diverges because

.

(**ii)** Geometric series allows us to express any repeating decimal as a fraction. For example,

.

The series is geometric with . Thus the series converges (since

). Since the first term is , it follows that .

**COMBINATION OF SERIES**

**Theorem:**

1. If and converge, then also converges and

.

1. If converges and , then also converges, and *.*
2. If diverges and , then also diverges. For example, diverges.
3. If converges and diverges, then is divergent.For example, diverges.
4. If both are divergent, then may or may not be convergent.

For example, if , then which is divergent. But, if and, thenwhich is convergent.

**Example:** Show that the series converges, and find its sum.

**Solution:** Since the series is geometric, we have , and we know that . This implies that . Therefore, the series

converges and .

**Exercise:**

1. Find a formula for the partial sums of the series. For each series determine the partial sums have a limit. If so, find the sum of the series.
2.
3.
4. Express the repeating decimal as fraction
5. …
6. …
7. …
8. …
	1. **A NONNEGATIVE TERM SERIES**

A series whose terms are nonnegative is called **nonnegative term series**. The partial sums of a nonnegative term series form an increasing sequence. That is,

 for . Thus if is bounded, then exists. This implies that converges. By contrast, if is unbounded, then cannot exist. This implies that diverges.

* 1. **CONVERGENCE TEST FOR NON NEGATIVE(POSITIVE ) TERM SERIES**
1. **INTEGRAL TEST**

**Theorem:** Let be a nonnegative sequence, and let be a continuous and decreasing

function defined on such that for . Then the series converges if

and only if the improper integral is converges. That is,

1. If converges, then the series converges.
2. If diverges, then the series diverges.

**Example:**Show that the series diverges.

**Solution:**Let for . Then is continuous and decreasing on , and for .

Now,

.

This implies that diverges. Hence, by the integral test, the series diverges.

**Example:** Show that the p-series converges.

**Solution:** If, then. This implies that diverges.

If , then which we know diverges (harmonic series).

Assume that . Let for . Then is continuous and decreasing on and for .

Now, .

If , then exists, and if , then does not exist.

Hence, converges if , and diverges if . Therefore, the series converges if , and diverges if .

For example, the series converges; and the series diverges.

**Note:** We should not inter from the integral test that the sum of the series is equal tothe

value of the integral.

In factwhereas. Therefore, in general,.

**Example:** Determine whether the series converges or diverges.

**Solution:** The function is nonnegative and continuous for. But it is not

obvious whether is decreasing or not. So we compute the derivative . Thus

 when . It follows that is decreasing when . And so we apply theintegral

test.

. Since the improper integral diverges, then the series also diverges by the integral test.

**Example:** Determine whether the series is convergent or divergent.

**Solution:**Let . We see that is continuous, decreasing &nonnegative for .So we apply the integral test.Hence,

.

.

Since we can use the L’Hopitals rule and we obtain .

Therefore, and so the series is convergent by integral test.

**Exercise:** Determine whether the given series are convergent or divergent using integral test.

1.
2. **COMPARISON TEST**

**Theorem:** Suppose that are series with nonnegative terms.

1. Ifconverges and for all , then converges, and .
2. If diverges and for all , then diverges.

**Example:** Determine whether the following series are convergent or divergent.

**Solution:**

1. Observe that for all and converges since is a -serieswith. Therefore, by comparison test theorem converges.

**b** &**c)**: **Exercise**

**Example:**Show that the series converges.

**Solution:** Here we notice thatfor . But we notice thatconverges. This means that also converges. So by the comparison test the given series converges.

1. **LIMIT COMPARISON TEST**

**Theorem:** Let be two series of nonnegative terms. Suppose

, where is a positive number.

1. If converges, then converges.
2. If diverges, then diverges.

**Example:** Test whether the following series are convergent or divergent.

**Solution:**

1. Let . Now, take , then

 and is a convergent -series (with ). Hence, by the limit comparison test **(i)**, the series converges.

1. **Exercise**

**Exercise:** Use the integral test, the comparison test or limit comparison test to determine whether the series converges or diverges.

1. **RATIO TEST**

**Theorem:** Let be a nonnegative series. Assume thatfor all and that

 (possibly ).

1. If, then converges.
2. If , then diverges.
3. If, then from this test alone we cannot draw any conclusion about the convergence or divergence of.

**Example**: Show that the seriesconverges.

**Solution**: Let . Then .

So, Since , the series converges.

**Example**: Show that the series diverges.

**Solution:**Exercise

1. **ROOT TEST**

**Theorem:** Let be a nonnegative series and assume that

 (possibly ).

1. If , then converges.
2. If , then diverges.
3. If , then from this test alone we cannot draw any conclusion about the convergence or divergence of .

**Example**: Show that

1. converges
2. diverges

**Solution:**

1. Taking the roots of the terms of the series, we get

. Thus, the root test implies that the seriesconverges.

1. . This implies that the series diverges by root test.

**Note:**Ratio Test is likely to be effective when the factorials or powers appear in the terms of the series, whereas Root Testis likely to be effective when powers (and not factorials) appear in the terms of the series. So, Ratio test is more frequently used than the Root Test.

**Exercise:** Determinewhether the given series are convergent or divergent.

* 1. **ALTERNATING SERIES AND ALTERNATING SERIES TEST**

If the terms in a series are **alternately positive and negative**, we call the series alternating

series. For example, the series

and

are alternating series.

**Theorem: (Alternating series test)**

If be a **decreasing sequence of positive** numbers such that . Then the alternating series and converge.

**Example:**Show that

1. converges
2. converges

**Solution:**

1. Since is a decreasing, nonnegative sequence and , the series satisfies the hypothesis of the alternating series test. Hence, the series converges.
2. **Exercise**
	1. **ABSOLUTE AND CONDITIONAL CONVERGENCE**

**Theorem:** If converges, then converges.

**Example:**Show that converges.

**Solution:** Since for**,** and since converges (because it is a -series with ), it follows that by the comparison test converges. So, by the above theorem, the given series also converges.

**Definition:** Let be a convergent series.

1. If converges, we say that the series converges absolutely.
2. If diverges, we say that the series converges conditionally.

**Examples:(i)** The series converges absolutely.

**(ii)** The series converges conditionally.

**Note:** All convergent nonnegative term series converge absolutely.

* 1. **GENERALIZED CONVERGENCE TEST**

**Theorem:** Let be a series.

1. **Generalized Comparison Test:**

If for , and if converges, then converges (absolutely).

1. **Generalized Limit Comparison Test:**

 If where is a positive number, and if converges, then converges (absolutely).

1. **Generalized Ratio Test:**

Suppose that for and that).

* If , then converges (absolutely)
* If , then diverges.
* If , then from this test alone we cannot draw any conclusion about the convergence of the series.
1. **Generalized Root Test:**

Suppose that for and that ).

* If , then converges (absolutely).
* If , then diverges.
* If , then from this test alone we cannot draw any conclusion about the convergence of the series.

**Example**:Show that

1. Converges absolutely for;
2. Converges conditionally for; and

**(iii)**Diverges for and for.

**Solution:** If , then the series obviously converges.

If , then

.

Therefore, the generalized ratio test implies that the series converges absolutely for and diverges for.

For, the series becomes the harmonic serieswhich diverges.

For , the series becomes which converges (alternating series).Since

 which diverges, we conclude that converges conditionally.

**Example**: Show that.

1. Converges absolutely for ;
2. Converges conditionally for ; and

**(iii)**Diverges for .

**CHAPTER-2: POWER SERIES**

* 1. **DEFINITION OF POWER SERIES AT ANY and.**

A power series (about or centered at is a series of the form

 (1)

where is a variable and the are constants called the **coefficient** of the series.

If , we obtain a power series (centered at )

 (2)

**Example:**are power series.

**Note:**Every power series defines a function whose domain is the collection of those of for which the power series converges.

**Example:** For what values of x the following series converges?

**Solution:**

1. Let . If , then

.

By the generalized ratio test, the series diverges when . Thus the given series converges only when .

1. Let . Then

.

By the generalized ratio test, the given series is absolutely convergent (and therefore convergent) when and divergent when .

Now, , so the series converges when and diverges when or .

The ratio test gives no information when , so we must consider and separately.

If , the series becomes which is divergent. If , the series is which converges. Thus the given power series converges for .

* 1. **CONVERGENCE AND DIVERGENCE, RADIUS AND INTERVAL OF CONVERGENCE**

**Theorem:** For a given power series , there are only three possibilities:

1. The series converges only when ;
2. The series converges for all ;
3. There is a positive number such that the series converges if and diverges if .

The number in case (iii) is called the **radius of convergence** of the power series.By convention, the radius of convergence in case (i) and in case (ii).

The **interval of convergence** of a power series is the interval that consists of all values of

for which the series converges. In case (i) the interval consists of just a single point ; in case

(ii)the interval is, and in case (iii) the interval is one of ,

.

**Remark:** When , anything can happen- the series might converge at one or both

endpoints, or it might diverge at both endpoint.

**Example:** We summarize here the radius and interval of convergence for each of the

examples already considered as follows.

|  |  |  |
| --- | --- | --- |
| **Power Series** | **Radius of Convergence** | **Interval of convergence** |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

In general, the Ratio Test (or sometimes the Root Test) should be used to determine the radius of convergence .

The Ratio Test and Root Test always fail when is an endpoint of the interval of convergence. So, the endpoint must be checked with some other test.

**Example:** Find the radius of convergence and interval of convergence of the following series

**Solution:**

1. Let . Then

.

By the generalized ratio test, the given series converges if and divergesif . Thus it converges and diverges if . Thus the radius of convergence is . This implies that the series converges in the interval . But we must now test for convergence at the endpoint of the interval.

If , the series becomes which diverges (observe that it is a p-series with or use integral test).

If , the series is which converges when . So, the interval of convergence is .

1. Let . Then

.

By the generalized ratio test, the given series converges if and diverges if . So, it converges and diverges if . Thus the radius of convergence is . The inequality can be written as . So, we test the series at the endpoints and .

When, the series is which diverges by divergence test ().

When , the series is which also diverges by divergence test. Thus the series converges only when when . So, the interval of convergence is .

**Exercise:** Find the radius of convergence and interval of convergence of the series

* 1. **REPRESENTATIONS OF FUNCTIONS AS POWER SERIES**

Here we represent certain types of functions as sums of power series by manipulating geometric series or by differentiating or integrating such a series.

Since is a geometric series with and then

 when . (**\***)

Therefore, the function is expressed as a sum of a power series when .

**Example:**Express as the sum of a power series.

**Solution:** Replacing by in equation (**\***), we have

.

Therefore,.

**Example:** Express as the sum of a power series and find the interval of convergence.

**Solution:** Replacing by - in equation (**\***), we have

.

Since this is a geometric series with and , it converges when).

Therefore, the interval of convergence is .

**Example:** Find a power series representation and interval of convergence forthe series

**Solution:**

1. To put this function in the form of equation (**\***), we first factor from the denominator

.

This series converges when i.e., . So the interval of convergence is

1. .

. So the interval of convergence is

* 1. **DIFFERENTIATION AND INTEGRATION OF POWER SERIES**

The sum of a power series is a function whose domain is the interval of convergence of the series. We would like to differentiate and integrate such functions, and the following theorem says that we can do so differentiating or integrating each individual term in the series. This is called **term-by-term differentiation and integration**.

**Theorem: (Differentiation and integration theorem for Power Series)**

If the power series has radius of convergence, then the function defined by is differentiable (and therefore continuous) on the interval and

1. .

The radii of convergence of the power series in equation (i) and (ii) are both .

**Note:**

1. Equations (i) and (ii) in the above theorem can be rewritten in the form
2. .
3. The radius of convergence remains the same when a power series is differentiated or integrated, this does not mean that the interval of convergence remains the same. It may happen that the original series converges at an endpoint, whereas the differentiated series diverges there.

**Example**: Express as the sum of a power series

Solution: We know that

Differentiating each side of the equation, we have

We can replace n by and write the answer as

**Example –8:** Show that 

Solution: The series  converges for all . The Differentiation Theorem tells us that  converges as well and that 

We see that for all real values of . The function satisfies the differential equation

From we see that

[Remember that we take even when for convenience in writing the general term]

 i.e. and we have the desired result.

Moreover, for all real values of

The radius of convergence is the same as the original series. i.e. R = 1.

**Theorem-3: (Integration Theorem for Power Series)**

Let  be a power series with radius of convergence 

Then  has the same radius of convergence, and



**Example –9:** Show that 

**Solution:** 

Replacing by – t in this equation, we obtain



Hence 

**Example –10**: Show that 

**Solution:** If  Therefore since we have

 

Then the Integration Theorem yields



In expanded form the power Series 

**Remark:** The radius of convergence remains the same when a power series is differentiated or integrated, this does not mean that the interval of convergence remains the same. It may happen that the original series converges at an endpoint, whereas the differentiated series diverges there.

**Exercise-3: 1.** Express the following functions as the sum of a power series**.**

1. b. c. d.

2. Find a power series expansion for and use it to evaluate.

3. Evaluate the following indefinite integral as a power series.

 a. b.

* 1. **Taylor Series, Taylor Polynomial and Application**

Let be the function defined by

 .

Let’s try to determine what the coefficients must be in terms of function .

If we put in equation (1),

Successive differentiation of the function in (1) and substitute of in equation gives.

If we continue to differentiate and substitute, we obtain

Solving this equation for the nth coefficient we get

 **. . . (\*)**

**(\*)** is valid even for

**Theorem-4:** If has a power series representation (expansion) at a, that is,

 if then its coefficients are given by the

 formula

**=**

and this series is called the **Taylor Series** of the function (or about or centered at ).

For the special case the Taylor series becomes

nd this series is called the **Maclaurin Series.**

* + 1. **Polynomial Approximation**

Define a polynomial of degree (at most n) by the formula

 =

This polynomial is called the Taylor polynomial of at

It is possible to measure the accuracy of this polynomial approximation.

Given Let

 **=** remainder (error) made in approximating.

**Example –11:** Construct the Taylor polynomial for 

Then determine a. and b.

**Solution:** and  for all n.

Thus,

= =

As a result, =

For = 

We expect to approximate . Since the value of is 2.71828 (accurate to six digits) and since . We find that approximates with an error of about

**Example –12:**Find the Taylor polynomial of degree 5 for at .

,

Therefore,

 =

**Theorem-5: (Taylor’s Inequality)**

If on some interval I containing for some constant M, then

 for all in .

**Note:** If then the Taylor series of the function converges to

**Example -13:** Find the Maclaurin series for and prove that it represents for all

**Solution:** ,

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as follows.

Hence 

Since , we have 1, for all . i.e.

Hence,

But If .

It follows from the sandwich theorem that and therefore, for all values of .

Therefore, is equal to the sum of its Maclaurin series. That is, for all



Similarly  for all

**Example-14:** Find the Maclaurin series for is any real number and.

**Solution:**

Thus, the Maclaurin series of is

Hence, and it is called **Binomial series**.

If its term is , then

If

 =

Thus, by the generalized ratio test, the binomial series converges if and diverges

**Theorem-6(Binomial series):** If is any real number and , then

**Example-15: a.**

**Example-16:** Find the Maclaurin series for the function and its radius of convergence.

**Solution:**

Using the binomial series with and with replaced by we have

 =

=

From the binomial series, the series converges when . i.e

Thus, the radius of convergence is

**Example-17: a.** Approximate the function by a Taylor polynomial of degree 2 at

b, How accurate is this approximation when (Use Taylor’s inequality).

 **Solution: a.**

Thus, the second degree Taylor polynomial is

 =

Hence, the desired approximation is

**b**, Using the Taylor’s inequality with and , we have

 where

Since we have and so

Hence, take

Also

Then the Taylor inequality gives

Thus, if then approximation in part (a) is accurate to within 0.0004.

**Exercise-4:**

1. Find the Maclaurin Series for f(x) and find the associated radius

of convergence.

 a b. c.

2. Find the Taylor Series for f(x) centered at the given values

 a. b.

3. i. Approximate f by a Taylor polynomial with degree n at the number .

 ii. Use Taylor in equality to estimate the accuracy of the approximation when

 x lies in the given interval.